Early Retirement Extreme

A philosophical and practical guide to financial independence

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Upfront Books

TABLE 7.1

SIMPLE SAVINGS RATE CALCULATIONS WITHOUT INTEREST RATE

If you work with a savings rate of $r\%$ for one year	You need to work this many years to take one year off
1%	99
5%	19
10%	9
15%	5.66
20%	4
25%	3
30%	2.33
40%	1.5
50%	1

	You can take this many years off before you need to work again
50%	1
60%	1.5
75%	3
80%	4
90%	9

7.3.2 FINANCIAL INDEPENDENCE AND INVESTING

The above calculations don't take investment returns on the fund into account. It stands to reason that if someone accumulates a 9 year fund by saving 75% while working for 3 years and invests it with a 5% return, it'll last longer than 9 years before it runs out, because of the investment returns on the money that hasn't been spent yet.

Presuming the returns can be guaranteed—more about that later—it's possible to calculate exactly how long such a fund will last if it compounds interest at a rate i. Suppose the fund has a size P_0 and each year p is withdrawn on the first day of the year, while the rest of the money is invested for a year at a rate i. Then the amount of money after one year will be

$$P_1 = (P_0 - p) + i(P_0 - p) = (P_0 - p)(1+i) = P_0(1+i) - p(1+i). \quad (7.1)$$

Another p is withdrawn (we have now withdrawn a total of $2p$) and the remaining amount $P_1 - c$ is invested again at the rate i. The amount available at the end of the second year is then

$$\begin{aligned} P_2 &= (P_1 - p)(1+i) = P_1(1+i) - p(1+i) \\ &= \big(P_0(1+i) - p(1+i)\big)(1+i) - p(1+i) \\ &= P_0(1+i)^2 - p(1+i)^2 - p(1+i), \end{aligned} \quad (7.2)$$

where we substituted in P_1 from the first equation. Repeating this, we find the remaining amount after the third year to be

$$P_3 = (P_2 - p)(1+i) = P_0(1+i)^3 - p(1+i)^3 - p(1+i)^2 - p(1+i) \quad (7.3)$$

and so on up to

$$\begin{aligned} P_N &= P_0(1+i)^N - p(1+i)^N - p(1+i)^{N-1} - \ldots - p(1+i)^2 - p(1+i) \\ &= P_0(1+i)^N - p\big[(1+i)^N + (1+i)^{N-1} + \ldots + (1+i)^2 + (1+i)\big]. \end{aligned} \quad (7.4)$$

Write the term in the square bracket as (S for sum)

$$S = (1+i)^N + (1+i)^{N-1} + \ldots + (1+i)^2 + (1+i), \quad (7.5)$$

then

$$(S+1)(1+i) = (1+i)^{N+1} + (1+i)^N + \ldots + (1+i)^2 + (1+i), \quad (7.6)$$

and so $(S+1)(1+i) - S = S + Si + (1+i) - S = Si + (1+i) = (1+i)^{N+1}$ because all the individual terms in the sum cancel out (set N to any random number and write the sum out to verify if in doubt) leading to

$$S = \frac{(1+i)^{N+1} - (1+i)}{i} = \frac{(1+i)((1+i)^N - 1)}{i}. \quad (7.7)$$

Substitute Equation 7.5 and Equation 7.7 back into Equation 7.4 to get

$$\begin{aligned} P_N &= P_0(1+i)^N - pS = P_0(1+i)^N - pS \\ &= P_0(1+i)^N - p\frac{(1+i)((1+i)^N - 1)}{i}. \end{aligned} \quad (7.8)$$

We are interested in using this formula to determine how long the portfolio will last, namely, how large is N?[5] When money runs out, $P_N \equiv 0$, therefore we rewrite Equation 7.8 as

$$0 = \frac{P_0}{p}i - (1+i)\left(1 - \frac{1}{(1+i)^N}\right), \quad (7.9)$$

and so

$$N = \log\left[\frac{1}{1 - \frac{P_0}{p}\frac{i}{1+i}}\right] \Big/ \log(1+i). \quad (7.10)$$

From this formula—which incidentally is the point where you can wake up again if you fell asleep during the derivation—we see that if we have a $P_0 = 10$ year fund which pays out $p = 1$ annually at a 4% ($i = 0.04$) interest, it lasts $N = 12.38$ years rather than 10 years. In comparison, a 20 year fund with the same parameters lasts 37.39 years. This is highly interesting, because by doubling the savings before beginning the withdrawal, an additional 15 years was gained on top of the 2.38 years from the 10 year fund's interest.

Equation 7.10 is the key formula for extreme early retirement, so pay extra attention to this paragraph! The formula relates the number of retirement years (your life expectancy upon retiring) to the rate of return on your portfolio and the size of your portfolio given in either withdrawal rates p/P_0 or the equivalent "years of fund" as illustrated by Table 7.1. Note that these two numbers are each other's exact inverse,[6] since if P_0 is given in years, then $p = 1$ year.

Figure 7.12 shows Equation 7.10, relating the size of the fund to how long it lasts for different values of i. First, note that $i = 0$ produces a straight line because if no interest is received, an N year fund will last exactly N years. Also, note that if $i = \frac{p/P_0}{1-p/P_0}$ then the denominator is 0 and $N \to \infty$. This means that the portfolio will last forever. The reason is that each year the portfolio grows exactly by the amount that is withdrawn. This is also called a perpetuity and it will preserve the principal forever. A quick rearrangement of $i = \frac{p/P_0}{1-p/P_0}$ yields

$$P_0 = (1 + i)p/i, \tag{7.11}$$

which is the required fund size to withdraw p at the beginning of each period when interest is added at the end of the period.[7] A perpetuity leaves an estate which is exactly equal to P_0. If less than p (as given by Equation 7.11) is withdrawn, then what is not withdrawn can be used to grow principal. This means that P will increase even as money is withdrawn.

The time to accumulate the fund P_0/p (given in years or months or whatever your preferred time unit is) can be calculated in a similar way. The simplest way to calculate it uses the method of Table 7.1

$$P_0/p = \frac{r}{1-r}M, \tag{7.12}$$

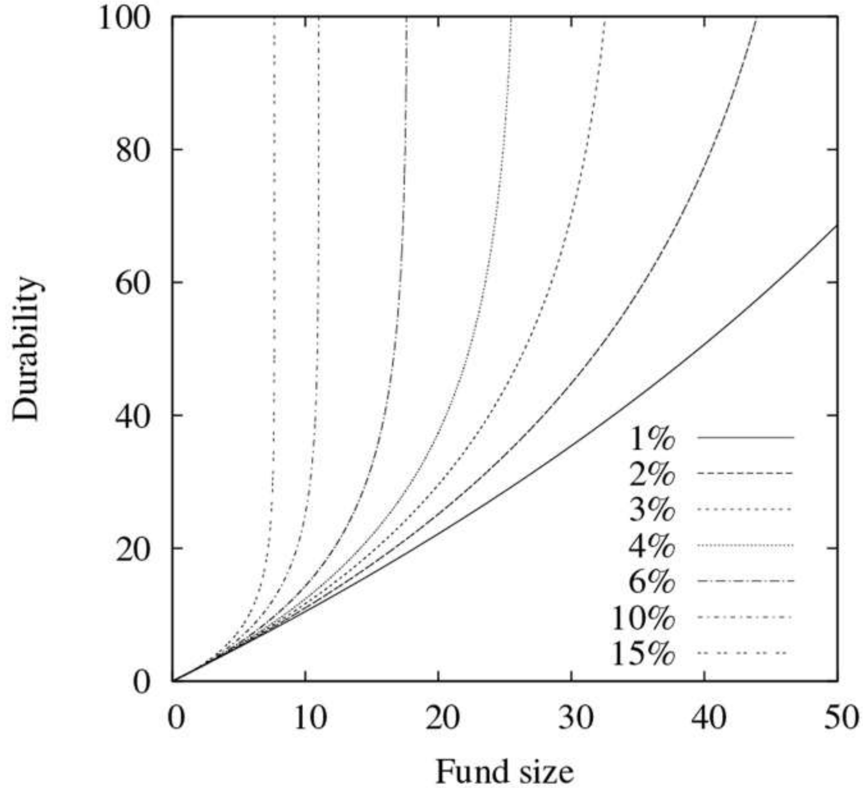

Figure 7.12: Size of fund given in years of expenses versus how long it will actually last before being depleted given different investment return rates.

where r is the savings rate and M is the number of years worked. This is the same equation that was used to generate Table 7.1. Now, if the funds are invested at a rate i and allowed to compound we get

$$P_0/p = \frac{r}{1-r}\sum_{i=1}^{M}(1+i)^{i-1}. \tag{7.13}$$

Note that this equation reduces to Equation 7.12 if $i = 0$. Using a similar trick to handle the sum, we find

$$P_0/p = \frac{r}{1-r}\frac{(1+i)^M - 1}{i}. \tag{7.14}$$

In traditional personal finance planning the time invested M is the most important factor, with typical values around 30 or 40 years. Some people will be clever enough to achieve superior investment returns i. We are not counting on this. In the case of early retirement, M will be small, and we will presume that i is of the usual range, maybe around 6%. The main lever in Equation 7.14 is thus $r/(1-r)$. For a traditional savings rate $r = 0.1$ the lever is $r/(1-r) = 0.11$, whereas for an extreme savings rate $r = 0.75$, the lever $r/(1-r) = 3$, which is 27 times higher! Even with a lot of time to compound and a market outperformance of a few percent, it's hard to beat a factor of 27.[8]

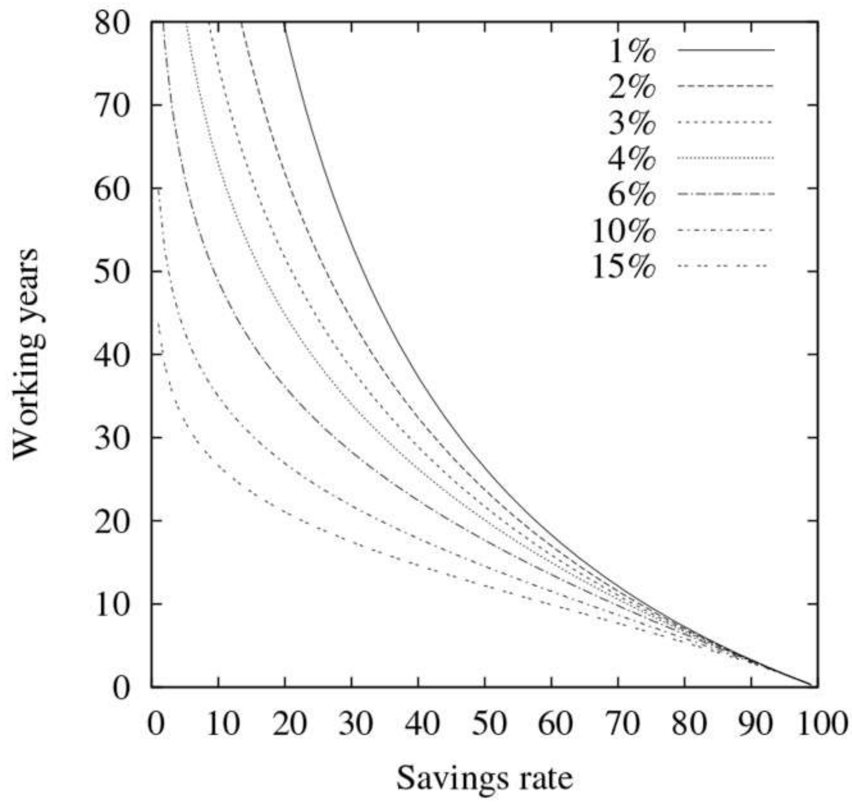

Figure 7.13: The time M it takes to grow P_0/p to 30 for a given savings rate r. A fund nominally lasting 30 years should be safe to retire on in most circumstances. Note that a traditional savings rate of 15% requires a little over 30 years of work to make it given average investment returns of 10%. If, on the other hand, investment returns drop to a conservative 6%, more than 45 years of work is required.

Figure 7.13 shows the time it takes to grow the fund to 30 years as a function of savings rate. According to Figure 7.12, a 30-year fund will last 70 years even at a modest 3% return rate.

Let us isolate M from Equation 7.14 to get

$$M = \log\left(1 + i\frac{P_0}{p}\frac{1-r}{r}\right) \Big/ \log(1+i). \tag{7.15}$$

Assuming that the life expectancy is 100 years and one is either financially independent or working to become so for the last 80 years of that time span (adjust your numbers as you see fit), then

$$80 = N + M, \tag{7.16}$$

where N is the number of years living off your money and M is the number of years spent accumulating it. We can now plot the working time M as a function of the savings rate r to find out how many years are needed to work to accumulate enough money for a given rate of return i. This is shown in Figure 7.14. This is the most important figure in this chapter. Specifically, it shows that high savings rates lead to extremely early financial independence. Conversely, the traditionally recommended savings rates mean working for 40 years or more, and they're very dependent on the return rate i. It also shows the difference between a savings rate of, say 35%, which most people would consider to be high, and a savings rate of 70%. It shows what your savings rate r should be to retire in M years. As is evident, saving three quarters of one's income creates financial independence in about five years! Conversely, a savings rate of 15% requires about 35 years of work at an optimistic return rate of 6%, about 20 years of work at a 10% rate, and about 45 years at a 4% rate.

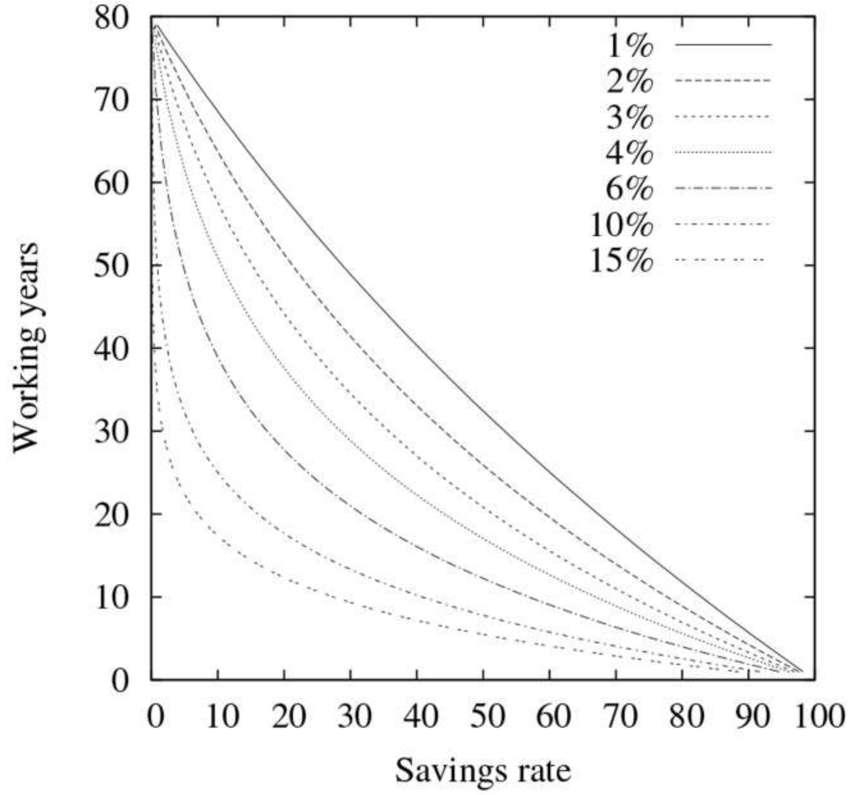

Figure 7.14: Figure showing savings rate against years to retirement for a range of investment return rates, presuming that the total number of years worked and years retired equals 80—for example, a person starts working at 20 and dies at 100.